

# Gravitational Shapiro phase shift on pulsars' period to detect Dark Matter

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August 11, 2013

## Abstract

The Shapiro Phase Shift on Pulsars (**SPSP**) influence at low , but detectable level the pulsars' (**PSR**) periods and their derivatives. This effect may be already written in PSRs' time derivative deviation and in particular it may explain the presence of a few negative pulse derivatives among 558 known PSRs. We describe in detail the phenomenon and predict that the **SPSP** may play a role in  $\dot{P}$  deviations for just a few PSRs a year ( $\sim 0.5 \div 5 \text{ } ev \text{ } yr^{-1}$ ) at a  $\dot{P} \sim (10^{-14} \div 10^{-15}) \text{ } s \text{ } s^{-1}$  level , leading to a plausible explanation to the puzzling negative pulse derivative of few PSRs' periods.

## 1 introduction

The search for gravitational microlensing events of stellar sources produced by MAssive Compact Halo Objects (**MACHOs**) provides an interesting but approximated tool for the study of dark matter in our galaxy. Although several events of achromatic magnification have been already observed , they have not let the MACHOs' parameters (mass , transversal velocity ...) be known independently. Comparisons of the observed optical depths with those provided by theoretical galactic models have just suggested that there must be more matter toward the galactic centre than in a galactic halo , making more probable that our galaxy is a barred one. Therefore the real nature of the deflector that produces an observed event remains still unclear. Unfortunately the possibility of studying interference between the two beams produced by a deflector fails , due to the incoherence of the stellar source.

Nevertheless the temporal phase shift produced by the deflector on “ monochromatic and coherent ” PSRs’ signals , the Shapiro effect , might provide another kind of investigation on the MACHO’s parameters. The extremely high precision the PSRs’ period can be known , and the explicit dependence of the time delay on the deflector’s mass , may offer a deeper tool for the measurement of both the deflector’s mass and impact parameter. An event of temporal phase gravitational shift , due to the massive object crossing the ray’s path between the PSR and the observer , produces the same phenomenon of displacing away and back the PSR on distances of the order of the Schwarzschild radius of the deflector. No Doppler shift (due to the gravitational pulling of a nearby crossing star) may lead to a similar forth-and-back phase shift , but it may only induce a one-time phase stretching. Doppler effects due to binary systems introduce a sinusoidal perturbation on residual PSRs’ signal of the same order of magnitude but with a different time signature and a lower probability.

A program of detection would require a continuous monitoring of the PSRs’ period because every event is a unique one and leaves no trace ; however it may hold for a period long enough to be observed.

We analyze the properties of this phenomenon by using the simple laws of gravitational lensing as found in literature. While the detectable gravitational magnification lensing needs millions of star sources (in order to fill up optical depths of  $\sim 10^{-7}$ ) , in our framework the Shapiro phase sensitivity (due to the logarithmic behaviour on the impact parameter) is much larger. This leads to larger detectable impact parameters , and so , larger optical depths (at least up to the needed  $\tau \sim 10^{-2}$  for the phenomenon to be observable with hundreds of PSR sources).

A similar effect between the two images produced by a microlensing event was already introduced [6] and it seems to be already observed on the PSR B0525+21 [7] : it is our belief that the associated optical depth be too low (see note at proof).

## 2 TIME DELAY VARIABILITY

Consider the gravitational lens [2] due to a MACHO passing by the line-of-sight of a PSR with transversal velocity  $v_{\perp}$  at a distance  $D_d$  from the observer. Although two images are produced by the lensing of the PSR , the secondary image is observable only when the MACHO enters within the microlensing tube in general defined by the threshold parameter  $u = 1$

[9, 10, 5] ; this means that the probability the secondary image be luminous enough is too low to take care of it , even if we assume at least  $\sim 500$  PSRs. Therefore from now on we shall consider only the primary image being observable.

The formula approximated (see the exact one in appendix A) that gives the time delay of the primary image's lensed ray relative to the undeflected ray , very similar to the Shapiro one [11] , depends on both a geometrical and a gravitational term :

$$\Delta t = \Delta t_{geo} + [\Delta t_{grav}] = \frac{r_S}{4c} \left( \sqrt{u^2 + 4} - u \right)^2 + \left[ \frac{r_S}{c} \log \left( \frac{8D_s}{r_S} \right) - \frac{2r_S}{c} \log \left( \sqrt{u^2 + 4} + u \right) \right] \quad (1)$$

where  $u$  is the adimensional impact parameter of the MACHO from the line-of-sight of the PSR in units of the Einstein radius :  $u = b / R_E$  (see figure 1) , where

$$R_E = \sqrt{2r_S \frac{D_d D_{ds}}{D_s}} = 4.84 \cdot 10^{13} \sqrt{\left[ \frac{M}{M_\odot} \right] \left[ \frac{D_s}{4Kpc} \right] \left[ \frac{x_d}{0.5} \right] \left[ 1 - \left( \frac{x_d}{0.5} \right) \right]} \text{ cm.} \quad (2)$$

$r_S$  is the Schwarzschild radius of the MACHO ,  $D_d$  ,  $D_s$  are respectively the observer-deflector and observer-source distances and  $D_{ds}$  the deflector-source distance. In the Einstein radius formula the substitutions  $x_d = D_d/D_s$  and  $1 - x_d = D_{ds}/D_s$  have been made.  $D_s$  is expressed in characteristic PSRs' distance units.

The geometrical term decreases very rapidly with the increase of  $u$  and it does not play any role as long as  $u \gg 1$  . The gravitational terms (a first one constant and a second one variable) account for most of the gravitational effect along the ray's path. The constant in the gravitational term underlines the positive behaviour of the time delay. The logarithmic divergence for  $u R_E \gg D_d$  ,  $D_{ds}$  , i. e. for  $u \rightarrow \infty$  , is fictious ; the time delay vanishes as  $\frac{r_S}{c} \frac{D_s}{b}$  , as shown in appendix A. As the formula shows , the time delay depends explicitly on the mass of the MACHO because of the presence of  $r_S$  ; so it gives more transparent and accurate informations on the lensing dark object than the measurement of any gravitational lensing magnification does ( $\propto 1/u$ ) at small  $u$ . Equation (1) is valid nearly always (as long as  $u \ll \sqrt{D_s/(2r_S)} \leq 10^8$  , for a characteristic solar mass) , namely for any realistic  $u \leq 10^3$ .

As the MACHO approaches to the line-of-sight of the PSR , the impact

parameter varies (in a first approximation) according to the law :

$$u(t) = u_{min} \sqrt{1 + \left(\frac{t}{t_c}\right)^2} \quad (3)$$

where  $u_{min}$  is the minimum impact parameter of the MACHO (figure 1) and

$$t_c = \frac{R_E}{v_\perp} u_{min} \approx 4.55 \frac{\left[\frac{u_{min}}{10^2}\right]}{\left[\frac{\beta_\perp}{10^{-3}}\right]} \sqrt{\left[\frac{M}{M_\odot}\right] \left[\frac{D_s}{4Kpc}\right] \left[\frac{x_d}{0.5}\right] \left[1 - \left(\frac{x_d}{0.5}\right)\right]} \text{ yrs} \quad (4)$$

is a characteristic time. The largest value of  $t_c$  is provided by a mass passing half-way between the observer and the PSR ( $x_d = 0.5$ ). A rough estimate of the integral phase residual time for our Shapiro shift in a  $10 t_c$  characteristic scale (an arbitrary cut-off time chosen for experimental data convenience) is :

$$\Delta t(u_{min}) - \Delta t(10 u_{min}) \approx 45 \left(\frac{M}{M_\odot}\right) \mu s \quad , \quad (5)$$

for large values of  $u_{min}$ . Detailed residual time evolutions from equation (1) are described in figures (2) and (3) for deflecting masses  $M_\odot$  and  $10^{-2} M_\odot$ . As we are allowing (on the contrary of the optical gravitational lensing of stars) large values of the impact parameter  $u$  we can neglect the first vanishing geometrical term in equation (1) and consider only the variable term in  $\Delta t_{grav}$ .

The speed of variation of the period  $P$  of the PSR is roughly the ratio of  $r_S/c$  and the characteristic Shapiro phase shift event time , namely its time evolution is :

$$\dot{P} = \frac{d \Delta t_{grav}}{dt} = \frac{d \Delta t_{grav}}{du} \frac{du}{dt} = \dot{P}_0 F(t) \quad (6)$$

where

$$\dot{P}_0 = \frac{2 r_S \beta_\perp}{R_E u_{min}} = \frac{1.38 \cdot 10^{-13} \left(\frac{\beta_\perp}{10^{-3}}\right) \sqrt{\frac{M}{M_\odot}}}{\left(\frac{u_{min}}{10^2}\right) \sqrt{\left(\frac{D_s}{4Kpc}\right) \left(\frac{x_d}{0.5}\right) \left[1 - \left(\frac{x_d}{0.5}\right)\right]}} s s^{-1} \quad (7)$$

$$F(t) = - \frac{\left(\frac{t}{t_c}\right)}{\sqrt{\left[1 + \left(\frac{t}{t_c}\right)^2\right] \left[1 + \left(\frac{t}{t_c}\right)^2 + \left(\frac{2}{u_{min}}\right)^2\right]}} \quad . \quad (8)$$

The considered  $\dot{P}_0$ -value for the present case , when the MACHO is half-way , has its minimum value. Therefore it defines a characteristic lower bound for residual PSRs' period variation. The function  $F(t)$  has a maximum and a symmetric minimum for the values  $\mp t_{max}$  , where :

$$t_{max} = t_c \left[ 1 + \left( \frac{2}{u_{min}} \right)^2 \right]^{\frac{1}{4}} \approx t_c , \quad (9)$$

(the approximation being valid for large values of  $u_{min}$ ) whose value (see equation 4) is nearly  $4.6 \sqrt{\frac{M}{M_\odot}} \text{ yr}$  . In figure (4) the function  $F(t)$  is plotted for the values  $u_{min} = 1$  and  $u_{min} \geq 10$  . It can be seen that as the MACHO approaches to the line-of-sight of the PSR (negative values of  $t$ ) , the PSR's period increases at a large rate , but , when  $u = u_{min}$  the growth decreases to zero ; the process is time-symmetric.

Using a double exponential disk model [1], the gravitational lensing optical depth can be evaluated. Choosing the galactic centre direction ( $l = 0^\circ$  ,  $b = 0^\circ$ ) , the calculation leads to the value

$$\tau = 2.33 \cdot 10^{-7} u_{min}^2 \left( \frac{\rho_0}{0.08 M_\odot pc^{-3}} \right) \left( \frac{D_s}{4 Kpc} \right)^2 \quad (10)$$

(being the integrated contribution of matter of the galactic bar component very low with respect to the galactic disk one between 4 and 8 Kpc from the galactic centre , it has been neglected). If this value is chosen as a reference one , both larger threshold parameters (of at least  $u_{min} = 10^2$ ) and the number of available PSRs ( $\geq 500$ ) can make the total optical depth be of the order of unity , i. e. a number

$$N \approx \tau N_{PSR} = 1.16 \left[ \frac{u_{min}}{10^2} \right]^2 \left[ \frac{N_{PSR}}{500} \right] \left[ \frac{\rho_0}{0.08 M_\odot pc^{-3}} \right] \left[ \frac{D_s}{4 Kpc} \right]^2 \quad (11)$$

of gravitational host MACHOs can always be found at the same time. A rough estimate of the rate of “complete” Shapiro phase shift events is :

$$\Gamma \approx \tau N_{PSR} \frac{v_\perp}{R_E u_{min}} \approx 0.51 \frac{\left( \frac{u_{min}}{10^2} \right) \left( \frac{N_{PSR}}{500} \right) \left( \frac{\beta_\perp}{10^{-3}} \right)}{\sqrt{\left( \frac{M}{M_\odot} \right) \left( \frac{D_s}{4 Kpc} \right)}} \text{ ev yr}^{-1} , \quad (12)$$

where in the last member the dependence on  $x_d$  has been eliminated by an exact volume integration. This rate estimate is based on known luminous lensing object galactic densities ; in presence of dark matter it may be one

or two orders of magnitude underestimated.

A competitive gravitational delay or enhancement in residual PSRs' periods may occur when an unbounded object (planet , star , black hole , ...) induce a gravitational deflection to the PSR (or the solar system) , leading to Doppler shifts (see appendix A1). Another equivalent shift occurs also for the presence of binary bounded objects near the PSR. Their influence induces a periodic sinousoidal residual signal in the PSR's period. Such an effect has been already observed at the level of intensity ( $\mu s$  over months) comparable to or below the one we are considering. However the signatures of such two different events are totally different (a step function increase or decrease , and a sinousoidal period residuals). The corresponding intensity and probability (related to the probability to find such an object in the surrounding volumes of the PSR or of the solar system) are extremely low with respect to the corresponding **SPSP** effect , whose probability is defined by much wider and longer cylindrical volume.

### 3 CONCLUSIONS AND FIRST EXPERIMENTAL EVIDENCES

The (**SPSP**) induces a unique characteristic secular variation on the PSRs' residual period : its signature leads to a pulse episodic bell-shaped time evolution of an order of tens of  $\mu s$  for one-solar-mass dark objects , well within the present detectability. It will never lead to an opposite sign event (first a decrease followed by an increase of the period). The pulse period variation could not be easily confused by a periodic Doppler shift , due to gravitational bounded planets , nor by rarest step increasing (or decreasing) Doppler shifts , due to an unbounded planet-star encounter near the PSR. The characteristic time residual  $\Delta t$  (equation 1) event , being dominated by a logarithmic function of the adimensional impact parameter  $u$  , has a wider cross section than gravitational lensing magnification and imply a larger probability to observe such an event even if at slow time scales. The characteristic scattering time is  $t_c \approx 4.6 \left( \frac{u_{min}}{10^2} \right) \left( \frac{\beta_1}{10^{-3}} \right)^{-1} \sqrt{\frac{M}{M_\odot}} \text{ yr}$  and the corresponding maximum amplitude in the period's derivative evolution  $|\dot{P}| = |F(t)| \dot{P}_0 \leq 6.9 \cdot 10^{-14} \left( \frac{u_{min}}{10^2} \right)^{-1} \left( \frac{\beta_1}{10^{-3}} \right) \sqrt{\frac{M}{M_\odot}} \text{ s s}^{-1}$  which is above or within the present instrumental sensitivity and it is even comparable to the present characteristic PSRs' period derivatives. The characteristic time evolution shown in figure (4) makes the effect observable and well defined. However

Table 1: PSRs that manifest negative  $\dot{P}$ .

| PSR        | $P$ (ms) | $\dot{P}$ ( $s s^{-1}$ ) | Environment |
|------------|----------|--------------------------|-------------|
| B 0021-72C | 5.7      | $-4.0 \cdot 10^{-17}$    | C           |
| B 1744-24A | 11.5     | $-1.9 \cdot 10^{-20}$    | B , C       |
| B 1813-26  | 59.3     | $-3.0 \cdot 10^{-16}$    |             |
| B 2127+11D | 4.8      | $-1.1 \cdot 10^{-17}$    | C           |
| B 2127+11A | 110.6    | $-2.1 \cdot 10^{-17}$    | C           |

In the last right column the notations mean : C = globular cluster , B = binary PSR

in the present catalogues of PSRs the period derivative  $\dot{P}$  has already an average positive derivative near  $10^{-14} s s^{-1}$ . This is related to the intrinsic PSR spin-down , due mainly to its luminosity radio emission. Therefore the Shapiro positive period derivative in earlier stages ( $\dot{P} > 0$ ) might be easily masked by common loss of angular momentum of the PSR. On the contrary a negative Shapiro period derivative near its maximum amplitude ( $|F| \approx 0.5$ ) in the second phase of the gravitational deflection (positive times , see figure 4) might overcome the positive PSRs' period derivative leading to an exceptional  $\dot{P}$ . For  $u_{min} = 10^2$  we must expect nearly  $0.5 ev yr^{-1}$  for  $M \sim M_{\odot}$  at an intensity  $|\dot{P}| \leq 6.9 \cdot 10^{-14} s s^{-1}$  and even more events ( $\sim 5 ev yr^{-1}$  for  $M \sim 10^{-2} M_{\odot}$ ) but at a lower level ,  $|\dot{P}| \leq 6.9 \cdot 10^{-15} s s^{-1}$  , almost on the edge of the characteristic intrinsic positive  $\dot{P}$ . Therefore lower-mass deflectors ( $M \leq 10^{-2} M_{\odot}$ ) will not easily perturbate the median intrinsic positive derivative. After a first inspection into the up-to-date catalogue of 558 PSRs [14] we do note the “anomalous” negative period derivatives shown in table 1. We note that the absolute  $\dot{P}$ -values are quite small , but assuming an average positive  $\dot{P} \sim 10^{-14} s s^{-1}$  we are well within our rough predictions ( $0.5 \div 5 ev yr^{-1}$ ). We also note that three PSRs (first , second and fourth ones) are found in globular clusters , where is more probable to find a star near the line-of-sight. However the globular cluster optical depth , even if additional to the external galactic one , is not large enough to change significantly the **SPSP** rate probability. Finally we note that 85 PSRs has unmeasured  $\dot{P}$  , possibly reflecting either intrinsic PSR stochastic behaviour or low level Shapiro phase shift noise. From figure (4) the maximum variability occurs near the maxima or the minima , where  $\dot{P} \leq 0$  . Any PSR

exhibiting the peculiarity  $\dot{P} \simeq 0$  ,  $\ddot{P} \leq 0$  (or in the fast transient linear phase when  $\dot{P} < 0$  ,  $\ddot{P} \simeq 0$  or , finally , in late stages when  $\dot{P} \simeq 0$  ,  $\ddot{P} > 0$ ) must be carefully kept under control. Their  $F(t)$ -evolution may lead to the expected double or unique bounce from its maximum or from its minimum. A careful analysis and monitoring on  $\dot{P}(t)$  evolution and deviation for all known PSRs (in particular the 85 whose  $\dot{P}$  variability is unknown and the above-mentioned five negative ones) may offer a unique tool to observe dark matter MACHOs. Because of the large impact parameter , the **SPSP** may observe also diluted dark molecular clouds which might be spread over a wide radius (  $10^{17} \text{ cm} > R > 10^{13} \text{ cm}$  ) and could not be easily observed by gravitational lensing magnification. In conclusion we do believe that **SPSP** is already acting on PSRs' periods (see also note at proof) and it may better probe the mysterious nature of dark matter in both the galactic disk and halo. First evidences from a few low ( $10^{-16} \div 10^{-17}$ )  $s \text{ s}^{-1}$  negative period derivative seem to favour light MACHO candidates of  $M \leq 10^{-1} M_{\odot}$  with respect to large ones  $M \geq M_{\odot}$ . Being the only one out of any cluster and any possible environmental disturbance , the PSR B1813-16 seems to be the best candidate suffering the **SPSP** process.

A possible mimic of the **SPSP** may derive from the exchange of angular momentum from an accreting disk near a PSR , leading to [12]

$$\dot{P} = -2.33 \cdot 10^{-12} \left( \frac{P}{\text{sec}} \right)^2 \left( \frac{\dot{M}}{10^{17} \text{ g s}^{-1}} \right)^{\frac{6}{7}} s \text{ s}^{-1} \quad (13)$$

for characteristic PSRs with accreting mass fluxes of  $\sim 10^{17} \text{ g s}^{-1}$ . However such a flux would be characteristic of a pulsating X-ray source ; moreover the presence of such a disk (even of lower flux) near a non-binary object is quite unexpectable : neutron stars are assumed to evacuate external volumes after their super-nova birth. The alternative (and extremely rare) capture , destruction (by tidal forces) of a planet would introduce , by keplerian eccentricity , a recognizable periodic Doppler modulation. Hypothetical but unknown secular angular momentum exchange (between external and internal fluids of the PSR) might also lead to such a period deviations. The “artificial” fine-tuned accretion of a circular disk near the PSR remains , however , the unique alternative possibility which we were able to argue.

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## A GRAVITATIONAL TIME DELAY

The gravitational time delay term is obtained by integrating along the ray's path the potential part of the effective refraction index introduced by the MACHO :  $\Delta t_{grav} = -\frac{2}{c^3} \int U dl = \frac{r_S}{c} \int \frac{dl}{r}$  [13]. The calculation led to the subsequent more general formula :

$$\Delta t_{grav} = \frac{r_S}{c} \sum_{i=1}^4 \log \left( a_i + \sqrt{a_i^2 + 1} \right) \quad (14)$$

where

$$a_1 = \frac{x_+}{D_d} \quad , \quad a_2 = \frac{x_+}{D_{ds}} \quad (15)$$

$$a_3 = \frac{D_d}{(b + x_+)} \left[ 1 - \left( \frac{b}{D_d} \right) \left( \frac{x_+}{D_d} \right) \right] \quad , \quad (16)$$

$$a_4 = \frac{D_{ds}}{(b + x_+)} \left[ 1 - \left( \frac{b}{D_{ds}} \right) \left( \frac{x_+}{D_{ds}} \right) \right] , \quad (17)$$

$x_+ = \frac{R_E}{2} \left[ \sqrt{u^2 + 4} - u \right]$  being the coordinate of the primary image (in the image plane having the origin towards the PSR, see figure 1),  $b = uR_E$  the impact parameter and, therefore,  $(b + x_+) = \frac{R_E}{2} \left[ u + \sqrt{u^2 + 4} \right]$ .

As long as  $u \rightarrow \infty$ ,  $a_1 \sim \frac{R_E}{uD_d}$ ,  $a_2 \sim \frac{R_E}{uD_{ds}}$ ,  $a_3 \sim \frac{D_d}{uR_E} - \frac{R_E}{uD_d}$  and  $a_4 \sim \frac{D_{ds}}{uR_E} - \frac{R_E}{uD_{ds}}$ . Therefore, as  $u$  tends to infinity  $\Delta t_{grav}$  fades to zero like  $\frac{r_S}{c} \frac{D_s}{b}$  (as it is expected).

Equation (14) is surely too sophisticated for our purposes. A simpler formula can be deduced from equation (14). In our limit  $b, R_E \ll D_d, D_{ds}$  ( $u \ll \sim \sqrt{\frac{2D_s}{r_S}}$ ,  $r_S \ll D_s$ ); therefore in this approximation only the third and fourth terms in equation (14) are important and, in both  $a_3$  and  $a_4$ , the second term in the square parenthesis is neglectable with respect to unity. The resulting formula is equation (1).

## A.1 THE DOPPLER SHIFT BY ENCOUNTERS OF NEARBY STARS

The possible momentum exchange due to an unbounded nearby object (star, planet, black hole, ...) in gravitational deflection leads to a tiny Doppler shift on the PSR's period. Let us assume a scattering parameter  $b$  (not to be confused with the above-considered **SPSP** line-of-sight impact parameter) by a one-solar-mass object of Schwarzschild radius  $r_{S_\odot}$ . The consequent orthogonal momentum transfer leads to an adimensional velocity exchange on the primordial PSR's motion is [3, 4]:

$$\langle \Delta \beta_{PSR} \rangle \simeq \left\langle \frac{r_{S_\odot}}{b} \frac{(1 + \beta_\odot^2)}{\beta_\odot} \right\rangle \leq 3.0 \cdot 10^{-9} \left( \frac{b}{0.1 \text{ ly}} \right) \left( \frac{\beta_\odot}{10^{-3}} \right)^{-1} \quad (18)$$

where  $\beta_\odot$  is the adimensional relative velocity between the PSR and the object. The corresponding average period derivative  $\langle \dot{P} \rangle_{Dp}$  during the whole time  $\Delta T$  of the gravitational deflection is

$$\langle \dot{P} \rangle_{Dp} \cong \frac{\Delta P}{\Delta T} \simeq \frac{\left( \frac{r_{S_\odot}}{b \beta_\odot} \right)}{\left( \frac{b}{\beta_\odot c} \right)} \approx 10^{-18} \cdot \left( \frac{P}{\text{sec}} \right) \left( \frac{b}{0.1 \text{ ly}} \right)^{-2} \left( \frac{M}{M_\odot} \right) s \text{ s}^{-1}. \quad (19)$$

This value which already assume a high and unprobable star density near a PSR ( $\geq 10^5 \text{ pc}^{-3}$ ) is negligible for most PSR's periods with respect to the characteristic **SPSP** delay.

In addition to the orthogonal momentum transfer , the parallel momentum transfer , in analogy to the electromagnetic bremsstrahlung , can reproduce also a bell-like **SPSP** time delay. If we observe the PSR with the line of sight in the same direction of propagation of the nearby crossing object (MACHO , Jupiters ...) , because of the exchange of parallel momentum , the PSR is sent away and back by , roughly , a distance  $\Delta l_{max} \approx \frac{GM}{\beta_{\odot}^2 c^2}$  in a characteristic time  $\frac{b}{\beta_{\odot} c}$  , producing a bell-like time delay whose maximum value is

$$\Delta t_{\parallel max} \approx \frac{r_s}{c} \frac{1}{\beta_{\odot}^2} . \quad (20)$$

Therefore the final figure will be a generic combination of the two effects (bell-like for parallel and step-like for orthogonal momentum transfer). We note that such bell-like PSR's period evolution may be sign-reversed because of the time-reversibility of the PSR-object scattering. In conclusion , out of a very rare geometrical configuration (in line of sight of the scattering direction) , the process should be in general distinguishable from our unique positive bell-like **SPSP** evolution.

## NOTE ADDED IN PROOF

We acknowledge the referee for letting us notice a previous article [7] where the **SPSP** have been applied to the residual period evolution of the PSR B0525+21 ( $D_s = 2.3 \text{ Kpc}$  ,  $l \approx 183^\circ$  ,  $b = 6.9^\circ$ ). The authors found out a bell-shaped evolution which is similar to the **SPSP** effect. However , even if their formulation of the integral phase residual time agrees with our

$\Delta t(u) - \Delta t(u_{min}) \approx -\frac{r_s}{2c} \log \left[ 1 + \left( \frac{t}{t_c} \right)^2 \right]$  (for large  $u$ ) , they did not discuss the optical depth for such a gravitational lensing event. From the fitting parameters of the deflecting mass (  $M \approx 330 M_{\odot}$  ) and characteristic evolution time ( $t_c \approx 0.32 \text{ yr}$ ) , equation (4) allows a minimum impact parameter of

$$u_{min} = \frac{0.51}{\sqrt{\left(\frac{x_d}{0.5}\right) \left[1 - \left(\frac{x_d}{0.5}\right)\right]}} \left( \frac{\beta_{\perp}}{10^{-3}} \right) . \quad (21)$$

As the most of the relative contribute  $\frac{1}{\tau} \frac{d\tau}{dx_d}$  to the optical depth is provided when the deflector is nearly half-way the distance  $D_s$  , we conclude

that  $u_{min} \leq 1$  . The theoretical optical depth for the PSR B0520+21 (nearly in the galactic anticentre direction) is  $\tau \leq 2.11 \cdot 10^{-8}$  . Due to this extremely low probability as well as the absence of the unavoidable twin lensing image ( $u_{min} \leq 1$ ) , we were forced to reconsider a slightly more probable explanation of the Doppler shift encounter (see appendix A1). Contrary to our earliest considerations for the orthogonal momentum transfer (see equations 18 , 19) , the parallel momentum transfer (equation 20) can reproduce a bell-like **SPSP** delay (with some additional step-like pattern).

We have imagined a free-free gravitational scattering and we noted that , if the largest delay must be  $\Delta t_{\parallel max} \approx 32 ms$  and if it is reached in nearly  $\frac{b}{\beta_{\odot} c} \approx 5yr$  , it might be produced by a mass of  $M \approx 10^{-3} \left( \frac{\beta_{\odot}}{10^{-3}} \right)^2 M_{\odot}$  with an impact parameter  $b \approx 4.5 \cdot 10^{15} \left( \frac{\beta_{\odot}}{10^{-3}} \right) cm$ . However a rough estimate of the probability for such a nearby encounter should be  $\approx 5 \cdot 10^{-7} \div 10^{-6}$  , respectively for realistic luminous or dark matter assumption ; still a small and puzzling value.

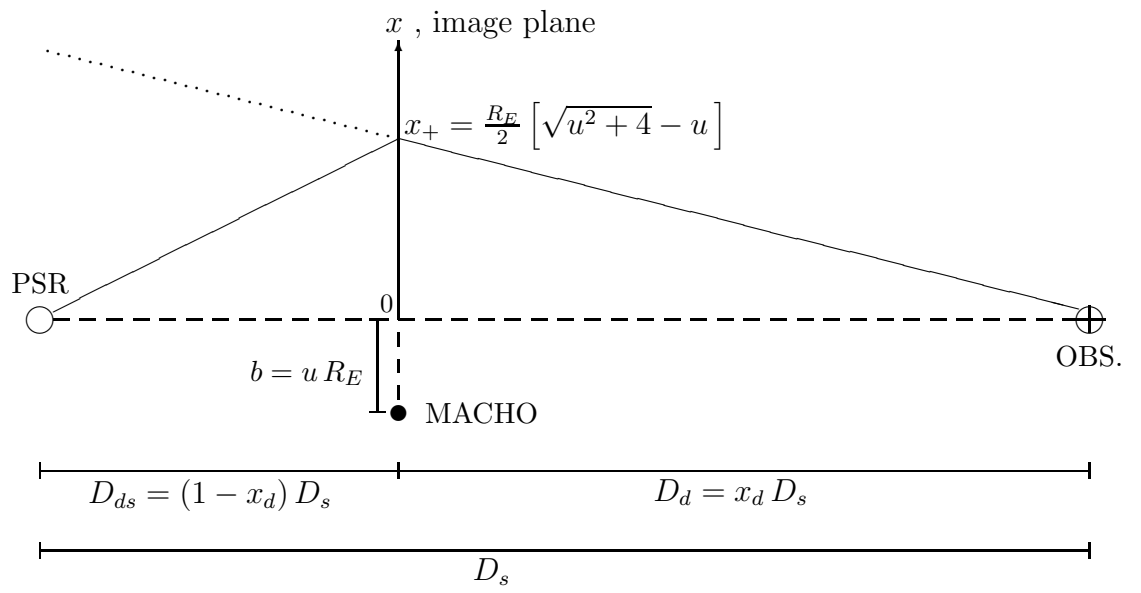


Figure 1: Relative position of source , MACHO , observer and primary image. As  $u \rightarrow \infty$  ,  $x_+ \rightarrow 0$  and the image corresponds to the source.  $v_\perp$  accounts for the relative PSR-MACHO-observer orthogonal motion velocity.